NONSTEADY CONJUGATE HEAT EXCHANGE DURING LIQUID FLOW IN COAXIAL CHANNELS

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The article deals with conjugate nonsteady heat exchange in coaxial channels during stabilized flow of a viscous incompressible liquid. The problem of the velocity profiles of the liquid is solved by methods of integral transformations. The conjugate problem of heat exchange in coaxial channels is solved by the finite-element method.

The flow of a liquid and heat exchange in channels formed by coaxial cylinders, one inside the other, is of interest in the calculation of various heat exchangers with forward flow and with counterflow. This is one of the examples when it is indispensable to solve the problem as conjugate so as to take account the heat transfer through the pipe walls with the smaller diameter from a liquid flowing through the annular channels to the liquid inside the cylindrical channel and vice versa. In analogy the problem can be solved for several coaxial channels. Heat exchange with the environment may also be regarded as conjugate if the conditions of heat exchange on the outer side are not specified, and the temperature fields in the environment can be investigated.

Here we will confine ourselves to the case of heat exchange specified on the outer surface of a pipe. We denote by r_1, r_2 and r_3, r_4 the outer and inner radii of the outer and inner pipes, respectively. We will take it that in the general case the thermophysical parameters of the liquids flowing through the inner cylindrical and the outer annular channels as well as the thermophysical parameters of the coaxial pipe are different.

The equations of motion and energy for liquids in channels are written in the form

$$\frac{\partial U_i}{\partial t} = -\frac{1}{\rho}\frac{\partial P_i}{\partial z} + \nu_i\left(\frac{\partial^2 U_i}{\partial r^2} + \frac{1}{r}\frac{\partial U_i}{\partial r}\right), \tag{1}$$

$$\rho_i C_{pi}\left(\frac{\partial T_i}{\partial t} + U_i\frac{\partial T_i}{\partial z}\right) = \lambda_i\left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r} + \frac{\partial^2 T_i}{\partial z^2}\right) + \Phi_i,\ i = 1,\ 3. \tag{2}$$

The equations of energy in the walls are

$$\rho_j C_{pj}\frac{\partial T_j}{\partial t} = \lambda_j\left(\frac{\partial^2 T_j}{\partial r^2} + \frac{1}{r}\frac{\partial T_j}{\partial r} + \frac{\partial^2 T_j}{\partial z^2}\right) + q_j,\ j = 2,\ 4. \tag{3}$$

We specify the initial and boundary conditions in the form

$$t = 0:\ U_i = U_{i0},\ T_i = T_{i0},\ T_j = T_{j0};$$

$$z = 0:\ T_i = T_{i0},\ T_j = T_{j0},\ U_i = U_{i0};$$

$$z \to \infty:\ \frac{\partial T_i}{\partial z} = \frac{\partial T_j}{\partial z} = \frac{\partial U_i}{\partial z} = 0;$$

$$\left.\begin{matrix}\varphi = 0\\ \varphi = \pi/2\end{matrix}\right\}:\ \frac{\partial T_1}{\partial \varphi} = \frac{\partial U_1}{\partial \varphi} = \frac{\partial T_2}{\partial \varphi} = \frac{\partial T_3}{\partial \varphi} = \frac{\partial U_3}{\partial \varphi} = \frac{\partial T_4}{\partial \varphi} = 0;$$

$$r = r_1:\ \lambda_1\frac{\partial T_1}{\partial r} = \lambda_2\frac{\partial T_2}{\partial r},\ T_1 = T_2,\ U_1 = 0;$$

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$$r = r_2:\ \lambda_2 \frac{\partial T_2}{\partial r} = \lambda_3 \frac{\partial T_3}{\partial r},\ T_2 = T_3,\ U_3 = 0;$$

$$r = r_3:\ \lambda_3 \frac{\partial T_3}{\partial r} = \lambda_4 \frac{\partial T_4}{\partial r},\ T_3 = T_4,\ U_3 = 0;$$

$$r = r_4:\ -\lambda_4 \frac{\partial T_4}{\partial r} = \alpha (T_4 - T_s),$$

where T_s is the ambient temperature.

We take as specified the pressure gradient

$$\frac{1}{\rho_i} \frac{\partial P_i}{\partial z} = f_i(r,\ t),$$

where $f_i(r, t)$ is a function whose form is determined by external conditions. This may be either a step function

$$f(r,\ t) = A\sigma(t - t_0),$$

where σ is Heaviside's unit function, which corresponds directly to the applied or relieved pressure, or a periodic function

$$f(t) = A \cos(\omega t - \varphi_0),$$

or else $f(r, t) = \text{const}$, etc.

On the hydrodynamically stabilized section the velocity distributions are specified by the expressions [1]:

$$U_{1_0} = 2\bar{U}_1 \left(1 - \frac{r^2}{r_1^2}\right), \tag{4}$$

$$U_{2_0} = 2\bar{U}_2 \frac{(r_2^2 - r^2) \ln \frac{r_1}{r_2} - (r_2^2 - r_1^2) \ln \frac{r}{r_2}}{r_2^2 - r_1^2 + (r_2^2 + r_1^2) \ln \frac{r_1}{r_2}}. \tag{5}$$

The problem of determining nonsteady velocity can be solved, e.g., by Laplace's or Hankel's integral transformations in finite limits since with the constant thermophysical parameters $\rho_i = \text{const}$, $\nu_i = \text{const}$ the problem is linear.

Let us examine how to find the velocity profile by Hankel's method of finite integral transformations [2, 3]

$$\bar{U}(s_m,\ t) = \int_0^{r_1} r K_\nu (s_m,\ r) U(r,\ t)\, dr, \tag{6}$$

where the kernel of the integral transformation K_ν is determined from the solution of the Sturm–Liouville problem for the Bessel equation

$$\frac{d^2\Phi_\nu}{dr^2} + \frac{1}{r} \frac{d\Phi_\nu}{dr} + \left(S_m^2 - \frac{\nu^2}{r^2}\right) \Phi_\nu = 0, \tag{7}$$

$$\Phi_\nu(r_1) = 0,\ \Phi_\nu(0) - \text{finite}. \tag{8}$$

Since the motion in the pipe does not depend on the angle ϕ, the azimuth number $\nu = 0$, and as solution of Eq. (7) we have to take the Bessel function of zeroth order

$$\Phi_0(s_m,\ r) = AJ_0(s_m,\ r) + BY_0(s_m,\ r), \tag{9}$$

where A and B are arbitrary integration constants; $J_0(s_m, r)$ is a Bessel function of the first kind; $Y_0(s_m, r)$ is a Bessel function of the second kind. From the boundary condition (8) we find B = 0, and from the second boundary condition and the solution of (9) we determine the spectrum of the eigenvalues as roots of the function J:

$$J_0(s_m, r) = 0.$$

The first n eigenvalues are given in [4]. It is more convenient to normalize the kernel of the integral transformation

$$K_0(s_m, r) = \Phi_0(s_m, r)/N, \tag{10}$$

where N is the norm determined by the relation

$$N^2 = \int_0^{r_1} rJ_0^2(s_m, r)\,dr = \frac{r_1^2}{2} J_0'(s_m, r_1).$$

Consequently, the kernel of the integral transformation is written in the form

$$K_0(s_m, r) = \frac{\sqrt{r}\,J_0(s_m, r)}{r_1 J_0'(s_m, r_1)}. \tag{11}$$

Then the inverse transformation is found in the form of the expansion

$$U(r, t) = \sum_{m=1}^{\infty} \frac{\sqrt{r}\,J_0(s_m, r)}{r_1 J_0'(s_m, r_1)} \overline{U}(s_m, t). \tag{12}$$

Applying Hankel's integral transformation with kernel (11) to Eq. (1), we obtain the following linear equation for determining the map $\overline{U}(s_m, t)$:

$$\frac{\rho_1}{\mu_1}\frac{d\overline{U}_1}{dt} + s_m^2 \overline{U}_1 = \frac{\overline{f}_1(s_m, t)}{\mu_1} + \left[r\left(K_0 \frac{dU_1}{dr} - U_1 \frac{dK_0}{dr}\right)\right]\Bigg|_0^{r_1}, \tag{13}$$

$$\overline{f}_1(s_m, t) = \int_0^{r_1} rK_0(s_m, r) f_1(r, t)\,dr,$$

$$t = 0:\ \overline{U}_{1_0} = \int_0^{r_1} rK_0(s_m, r) U_{1_0}(r)\,dr = \overline{U}_{1_0}(s_m), \tag{14}$$

where the last term vanishes because with r = 0 it is identically equal to zero, and with $r = r_1$, $U_1 = 0$. Incidentally, if there is slip on the boundary, i.e., with $r = r_1$, $U = U_1(t)$, then Eq. (13) assumes the form

$$\frac{\rho_1}{\mu_1}\frac{d\overline{U}_1}{dt} + s_m^2 \overline{U}_1 = \frac{f_1(s_m, t)}{\mu_1} - rU_1(t)\left.\frac{dK_0(s_m, r)}{dr}\right|_{r=r_1} \equiv \overline{F}_1(s_m, t).$$

The solution of the first-order equation with the right-hand side on condition (14) is found in the quadratures

$$\overline{U}_1(s_m, t) = \exp\left(-\frac{\mu_1}{\rho_1} s_m^2 t\right)\left[\overline{U}_{1_0}(s_m) + \int_0^t \exp\left(\frac{\mu_1}{\rho_1} s_m t'\right)\overline{F}(s_m, t')\,dt'\right].$$

Consequently, the problem of determining the map of the velocity $\overline{U}_1$ for any specified functions $U_1(t)$, $\overline{U}_{1_0}$, and f is solved in principle, and from it we find the original $U_1(r, t)$ by the formula

$$U_1(r, t) = \sum_{m=1}^{\infty}\left[U_{1_0}(s_m) + \int_0^t \exp\left(-\frac{\mu_1}{\rho_1} s_m^2 t'\right)dt'\right]\exp\left(-\frac{\mu_1}{\rho_1} s_m^2 t\right) K_0(s_m, r). \tag{14'}$$

Analogously we find the solution of Eq. (1) in the region between two coaxial cylinders with the aid of Hankel's integral transformation whose kernel is expression (10), where Φ_0 contains both Bessel functions (9) and satisfies the first boundary conditions at both ends

$$\Phi_0(r_2) = 0, \quad \Phi_0(r_3) = 0. \tag{15}$$

Using (15) and the solution of (9), we find the system

$$AJ_0(s_m, r_2) + BY_0(s_m, r_2) = 0,$$
$$AJ_0(s_m, r_3) + BY_0(s_m, r_3) = 0, \tag{16}$$

which has a solution if its determinant vanishes; this yields a transcendental equation for determining the eigenvalues

$$J_0(s_m, r_2)\, Y_0(s_m, r_3) = J_0(s_m, r_3)\, Y_0(s_m, r_2).$$

The roots of this equation were tabulated in [4], consequently the spectrum s_m of the eigenvalues is known. From (16) we find

$$B = -A\frac{J_0(s_m, r_2)}{Y_0(s_m, r_2)},$$

i.e.,

$$\Phi_0(s_m, r) = N(J_0(s_m, r)\, Y_0(s_m, r_2) - Y_0(s_m, r)\, J_0(s_m, r_2)).$$

The norm is determined from the expression

$$N^2 = \int_{r_2}^{r_3} r\Phi_0^2(s_m, r)\, dr =$$
$$= \frac{r_3^2}{2}[\Phi_0'^2(s_m, r_3) + \Phi_0^2(s_m, r_3)] - \frac{r_2^2}{2}[\Phi_0'(s_m, r_2) + \Phi_0^2(s_m, r_2)].$$

We apply Hankel's integral transformation with the kernel

$$K_0(s_m, r) = (J_0(s_m, r)\, Y_0(s_m, r) - Y_0(s_m, r)\, J_0(s_m, r_1))/N$$

to Eq. (1)

$$\int_{r_2}^{r_3} rK_0(s_m, r)\left[\frac{\partial^2 U_3}{dr^2} + \frac{1}{r}\frac{\partial U_3}{\partial r} + \frac{1}{\mu_3} f_3(r, t) - \frac{\rho_3}{\mu_3}\frac{\partial U_3}{\partial t}\right] dr = 0.$$

Integrating by parts and using the boundary conditions, we obtain the following equation of first order for determining $\overline{U}_3(s_m, t)$:

$$\frac{\rho_3}{\mu_3}\frac{d\overline{U}_3}{dt} + s_m^2\overline{U}_3 = \overline{F}_3(s_m, t), \tag{17}$$

where

$$\overline{U}_3\big|_{t=0} = U_{3_0}(s_m);$$

$$\overline{F}_3(s_m, t) = \frac{1}{\mu_3}\int_{r_2}^{r_3} f_3(r, t)\, rK_0(s_m, r)\, dr;$$

$$\overline{U}_{3_0} = \int_{r_2}^{r_3} U_{3_0}(r)\, rK_0(s_m, r)\, dr.$$

Taking the solution of (13)-(14) in the form of (14') into account, we obtain an expression for the velocity in the form an an expansion

$$U_3(r, t) = \sum_{m=0}^{\infty}\left[\overline{U}_{30}(s_m) + \int_0^t \exp\left(\frac{\mu_3}{\rho_3} s_m^2 t'\right)\overline{F}_3(s_m, t')\, dt'\right]\exp\left(-\frac{\mu_3}{\rho_3} s_m^2\, t\right) K_0(s_m, r).$$

This solution, suitable for the specified initial velocities U_{i_0} and requiring the external forces

$$-\frac{\partial P_i}{\partial z} = f_i(r, t),$$

entails certain difficulties when used in practice, therefore, regardless of the general nature of the obtained closed solution, it is often of interest to investigate particular simplified solutions.

Conjugate convective heat exchange in coaxial channels during hydrodynamically stabilized laminar flow of a liquid through them is described by the dimensionless system of equations (18)-(21) with the initial and boundary conditions:

$$\frac{\partial W_i}{\partial z}=0, \tag{18}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial W_i}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 W_i}{\partial \varphi^2}=-F_i, \tag{19}$$

$$\frac{\partial \Theta_i}{\partial \mathrm{Fo}_i}+\mathrm{Pe}_i W_i\frac{\partial \Theta_i}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta_i}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Theta_i}{\partial \varphi^2}+\frac{\partial^2 \Theta_i}{\partial z^2}, \quad i=1,\ 3, \tag{20}$$

$$\frac{\partial \Theta_j}{\partial \mathrm{Fo}_i}=\frac{a_j}{a_i}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta_j}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Theta_j}{\partial \varphi^2}+\frac{\partial^2 \Theta_j}{\partial z^2}\right),\quad j=2,\ 4, \tag{21}$$

$$\mathrm{Fo}_i=0:\Theta_i=\Theta_j=0;\quad z=0:\Theta_i=\Theta_j=0;$$

$$\left.\begin{matrix}\varphi=0\\ \varphi=\pi/2\end{matrix}\right\}:\frac{\partial W_1}{\partial \varphi}=\frac{\partial W_3}{\partial \varphi}=\frac{\partial \Theta_1}{\partial \varphi}=\frac{\partial \Theta_2}{\partial \varphi}=\frac{\partial \Theta_3}{\partial \varphi}=\frac{\partial \Theta_4}{\partial \varphi}=0;$$

$$r=r_1:W_1=0,\qquad \Theta_1=\Theta_2,\ \lambda_1\frac{\partial \Theta_1}{\partial r}=\lambda_2\frac{\partial \Theta_2}{\partial r};$$

$$r=r_2:W_3=0;\quad \Theta_2=\Theta_3,\quad \lambda_2\frac{\partial \Theta_2}{\partial r}=\lambda_3\frac{\partial \Theta_3}{\partial r};$$

$$r=r_3:W_3=0,\quad \Theta_3=\Theta_4,\quad \lambda_3\frac{\partial \Theta_3}{\partial r}=\lambda_4\frac{\partial \Theta_4}{\partial r};$$

$$r=r_4:\begin{cases}\Theta_4=\dfrac{T-T_0}{T_s-T_0}=1, & \text{first boundary conditions,}\\[2ex] \Theta_4=\dfrac{(T-T_0)\lambda_3}{cq_s}, & \text{second boundary conditions,}\end{cases}$$

where

$$W_i=U_i/U_{i_0};\quad F_i=\frac{\Delta P_i c}{U_{i_0}\mu_i};\quad \mathrm{Fo}_i=a_i t/c^2;$$

$$r=R/c;\quad z=z'/c;\quad \mathrm{Pe}_i=U_{i_0}c/a_i;\quad a_1=a_3;$$

$$a_2=a_4;\quad c=2R_4;\quad \mu_1=\mu_3;$$

$$\mathrm{Fo}_i\geqslant 0;\quad z\geqslant 0;\quad 0{,}0\leqslant r\leqslant 0{,}5;\quad 0\leqslant\varphi\leqslant\pi/2;$$

here, q_s and T_s are the specified heat flux and temperature, respectively, on the external surface of the wall of the annular channel; T_0 is the temperature at the inlet to the channel.

The heat carrier is regarded as homogeneous dropping liquid without proceeding chemical reactions, the range of temperature change is small so that the dependence of the thermophysical properties of the liquid and of the material of the wall on the temperature may be neglected. The pressure gradient along the channel axis is assumed to be constant and known.

The problem is solved by the joint application of the finite element method and the finite difference method. In accordance with the finite element method, the first stage consists in the discretization of the domain of the channel section, i.e., the nodal points and the so-called two-dimensional simplex elements (linear elements of triangular shape) are determined and numbered. The basic rules concerning discretization are discussed in detail in [5, 6].

At the subsequent stage the continuous magnitude $f(r, \phi, z, Fo)$ (temperature, velocity) is approximated by a discrete model which is constructed on the set of piecewise continuous functions $f^{(m)}(r, \phi, z, Fo)$ determined at the final number of subregions (elements). On each element the sought functions (temperature, velocity) are represented in the form [5, 6]

$$f^{(m)}(r,\ \varphi,\ z,\ \mathrm{Fo})=\sum_{k=1}^{3} a_k^{(m)}(z,\ \mathrm{Fo})\,\varphi_k^{(m)}(r,\ \varphi),$$

where m is the number of the element; $\phi_k^{(m)}(r, \phi)$ are functions of the shape (linear base) of the triangular element; $a_k^{(m)}(z, Fo)$ are the unknown nodal values of the sought function at the apexes of the element (in case of hydrodynamically stabilized flow $a_k^{(m)}(z, Fo) = const$).

To obtain a resolvent system of equations for determining the nodal values of velocity and temprature, the Bubnov—Galerkin method [7] is applied to the equations of motion and energy written for the m-th finite element. For instance, for the equation of energy in the heat carrier we obtain:

$$\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\,dV^{(m)}\frac{\partial\{\Theta'^{(m)}\}}{\partial \mathrm{Fo}}+\mathrm{Pe}\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\{W^{(m)}\}[N^{(m)}]\,dV^{(m)}\frac{\partial\{\Theta'^{(m)}\}}{\partial z}=$$

$$=\iint\limits_{V^{(m)}}[N^{(m)}]^*\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial[N^{(m)}]}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\frac{\partial[N^{(m)}]}{\partial\varphi}\right)\right]dV^{(m)}\{\Theta'^{(m)}\}+$$

$$+\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\,dV^{(m)}\frac{\partial^2\{\Theta'^{(m)}\}}{dz^2},$$

where $[N^{(m)}]^*$ is the matrix obtained by transposition of $[N^{(m)}]$; $[N^{(m)}]$ is the matrix of shape function; integration is carried out with respect to the domain of the finite element with the number $m(V^{(m)})$. We transform:

$$\iint\limits_{V^{(m)}}[N^{(m)}]^*\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial[N^{(m)}]}{\partial r}\right)\right]dV^{(m)}=$$

$$=\iint\limits_{V^{(m)}}\frac{1}{r}\frac{\partial}{\partial r}\left([N^{(m)}]^*r\frac{\partial[N^{(m)}]}{\partial r}\right)dV^{(m)}-\iint\limits_{V^{(m)}}\frac{\partial[N^{(m)}]^*}{\partial r}\frac{\partial[N^{(m)}]}{\partial r}\,dV^{(m)}.$$

Using the formula of Ostrogradskii—Gauss, we find

$$\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\frac{\partial\{\Theta'^{(m)}\}}{\partial \mathrm{Fo}}\,dV^{(m)}+\mathrm{Pe}\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\{W^{(m)}\}[N^{(m)}]\,dV^{(m)}\frac{\partial\{\Theta'^{(m)}\}}{\partial z}=$$

$$=-\iint\limits_{V^{(m)}}\left(\frac{\partial[N^{(m)}]^*}{\partial r}\frac{\partial[N^{(m)}]}{\partial r}+\frac{1}{r^2}\frac{\partial[N^{(m)}]^*}{\partial\varphi}\frac{\partial[N^{(m)}]}{\partial\varphi}\{\Theta'^{(m}\}\,dV^{(m)}+\right.$$

$$+\int\limits_{s^{(m)}}[N^{(m)}]^*\frac{\partial[N^{(m)}]}{\partial\bar{n}^{(m)}}\,ds^{(m)}\{\Theta'^{(m)}\}+\iint\limits_{V^{(m)}}[N^{(m)}]^*[N^{(m)}]\,dV^{(m)}\frac{\partial^2\{\Theta'^{(m)}\}}{\partial z^2},\tag{22}$$

where $s^{(m)}$ is the boundary of the m-th triangular element. Then we compose the sum [6]:

$$\Theta(r,\ \varphi,\ z,\ \mathrm{Fo})=\sum_{m=1}^{n}\Theta^{(m)}(r,\ \varphi,\ z,\ \mathrm{Fo}),\tag{23}$$

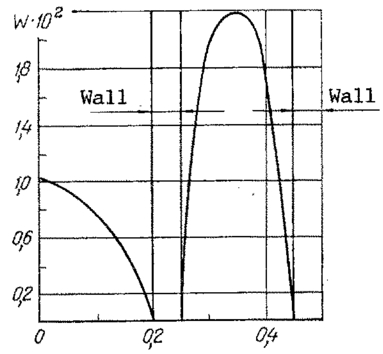

Fig. 1

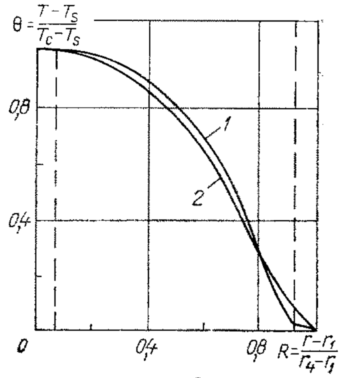

Fig. 2

Fig. 1. Velocity profiles in the annular and the cylindrical channel.

Fig. 2. Temperature distribution over the width of the annular channel: 1) curve obtained by the present authors; 2) curve after [1].

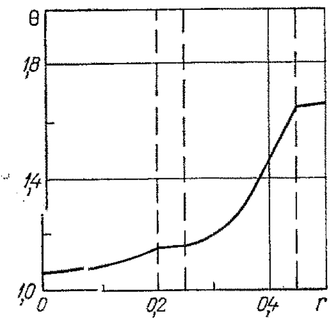

Fig. 3

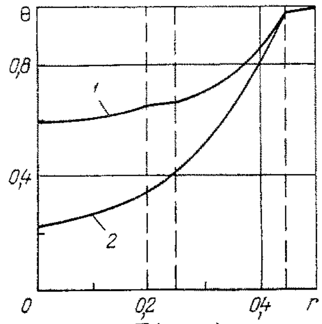

Fig. 4

Fig. 3. Temperature distribution over the width of a cylindrical coaxial channel for the second boundary conditions on the external wall.

Fig. 4. Temperature distribution in coaxial cylindrical channels (1) and in a cylindrical channel (2) with the first boundary conditions on the external wall.

where

$$\Theta^{(m)}(r,\ \varphi,\ z,\ \mathrm{Fo}) = [N^{(m)}(r,\ \varphi)]\{\Theta'^{(m)}(z,\ \mathrm{Fo})\},$$

n is the total number of triangular elements whose set constitutes the geometric discretization of the domain of the channel cross section. Applying (23) to Eq. (22), we have

$$A\frac{\partial\{\Theta\}}{\partial \mathrm{Fo}} + B\frac{\partial\{\Theta\}}{\partial z} = C\{\Theta\} + D\frac{\partial^2\{\Theta\}}{\partial z^2} + \{F\}. \tag{24}$$

In an analogous way we find the equation of motion of the liquid

$$\overline{B}\{W\} = \{P\}. \tag{25}$$

Here A, B, C, D, $\overline{B}$ are global strip-type matrices obtained by summing over all the elements of the respective matrices in Eqs. (24)-(25). For instance, $A = \sum_{m=1}^{n} A^{(m)}$, , where

$$A^{(m)} = \iint_{V(m)} [N^{(m)}]^* [N^{(m)}]\, dV^{(m)}, \quad \text{etc.}$$

The vectors {F} and {P} take into account the effect of the pressure forces in the flow and the boundary conditions on the outer channel wall surface, respectively.

The system of equation (24)-(25) is solved with the aid of the difference schema with approximation of the second derivative with respect to the z-coordinate by Saul'ev's schema [8]:

$$\frac{1}{\Delta \mathrm{Fo}} A\{\Theta\}_i^{n+1} + \frac{1}{\Delta z} B\{\Theta\}_i^{n+1} - C\{\Theta\}_i^{n+1} = \frac{1}{\Delta \mathrm{Fo}} A\{\Theta\}_i^n +$$

$$+ \frac{1}{\Delta z} B\{\Theta\}_{i-1}^{n+1} + \frac{1}{(\Delta z)^2} D[\{\Theta\}_{i-1}^{n+1} - \{\Theta\}_i^{n+1} - \{\Theta\}_i^n + \{\Theta\}_{i+1}^n] + \{F\}_i^{n+1},$$

where i is the number of the theoretical channel section; n is the number of the theoretical instant.

The following parameters were specified:

$$\mathrm{Pe} = 1000, \quad RT = a_j/a_i = 12, \quad F_1 = 1{,}0, \quad F_3 = 4{,}5.$$

The thickness of the channel walls changed:

$$1)\ r_1 = 0.20,\ r_2 = 0.25,\ r_3 = 0.45,\ r_4 = 0.50;$$
$$2)\ r_1 = 0.20,\ r_2 = 0.22,\ r_3 = 0.48,\ r_4 = 0.50.$$

With the second boundary conditions, the heat flux $q_s = 2.5 \cdot 10^4$ W/m^2 was specified on the outer wall of the annular channel. The results of the calculations are presented in Figs. 1-4.

Figure 1 shows the velocity profiles of the heat carrier on the hydrodynamically stabilized section of a cylindrical channel ($F_1 = 1$) and of an annular channel ($F_3 = 4.5$). It agrees well with the analytical expressions (12) and (13), respectively. The temperature distribution in coaxial channels with the first and second boundary conditions on the external wall surface of an annular channel is shown in Figs. 3 and 4. In Fig. 2, the theoretical temperature distribution in an annular channel with finite wall thickness ($r_4 - r_3 = 0.02$) is being compared with an analogous distribution after [1], with the wall thickness not taken into account. The dependence of the temperature distribution in a cylindrical channel on the Pe, RT, Bi numbers was dealt by us in a previous work [9].

NOTATION

U, velocity of the direction z; t, time; ρ, density; P, pressure; C_p, specific heat; T, temperature; λ, thermal conductivity; μ, dynamic viscosity; q, specific density of internal heat sources; Φ, dissipation function; K_ν, kernel of Hankel's integral transformation; N, norm of Hankel's integral transformation; $\bar{U}(s_m, t)$, map of velocity U(r, t); J_0, Bessel function of the first kind; Y_0, Bessel function of the second kind; W, dimensionless velocity in the direction z; c, diameter of the outer cylinder; Θ, dimensionless temperature; T_0, temperature at the channel inlet; T_s, q_s, temperature and heat flux, respectively, on the surface of the outer cylinder; a, thermal diffusivity; Fo, dimensionless time; r, ϕ, z, dimensionless cylindrical coordinates; ΔP, pressure increment; Pe, Peclet number; m, number of the finite element; $N^{(m)}$, shape function of the m-th finite element; $[N^{(m)}]$, row vector of the shape function of the element with number m; $\{\Theta^{(m)}\}$, nodal values of the velocity in the channel; $V^{(m)}$, domain of the finite element with the number m; Bi, Biot number; RT = a_j/a_i.

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SELF-SIMILAR HEATING REGIME UPON DESTRUCTION OF THE SURFACE OF MATERIALS

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The applicability of the dependence $\Delta^* \approx K\sqrt{a\tau}$ in mass transfer from the surface of heat insulating materials is experimentally demonstrated. A formula for calculating the temperature coefficient K is suggested.

In the classical theory of heat conduction the notion of self-similar heating is widely used; this means that a dimensionless Fourier number becomes the single variable determining the process of heat propagation. It is believed that to establish this regime, it is necessary that the temperature of the outer, heated surface be maintained constant and that mass transfer from the surface either by nonexistent, or that its rate be inversely proportional to the square root of the time.

However, the self-similar solution for a semiinfinite body not subject to destruction and with constant temperature $T_w = T_p$ = const [1]

$$\Theta^* = \frac{T^* - T_0}{T_w - T_0} = \operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right) \tag{1}$$

satisfies even more complex variants of thermal loading. For instance, according to the calculations by A. V. Vasin, when the surface temperature changes trapezoidally, the depth of heating δ_T is described by the "almost" self-similar expression

$$\delta_T \approx K\sqrt{a\tau}, \tag{2}$$

if $m/(e + n) \geq 2$. Here,

$$K = \Theta^{*-0.3}; \tag{3}$$

e, n are the heating and cooling sections, respectively, and m is the section with the temperature T_w = const.

In distinction to the classical self-similar regime, in the experiments of [2] a quasisteady velocity of surface mass transfer was observed. The time of establishing such a velocity was about one fifth of the time τ_δ, nevertheless, in the time interval $\tau \leq \tau_\delta$ the distance through which the isotherm of phase transformations passed obeyed the dependence type (2). In these experiments the quasisteady velocity of mass transfer changed to a multiple while the surface temperatures were practically equal. However, the overall amount of heated and removed material within the same time of heating remained the same within the accuracy of the experiment. Such conditions of destruction were attained by testing specimens in air and nitrogen plasma and under radiative heating. It may consequently be assumed that the regularity of change in velocity of the outer surface has no effect on the rate of displacement of the isotherm of phase transformations in the time interval $\tau \leq \tau_\delta$.

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